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Article

The Singapore method as a strategy for improving mathematics learning

El método singapur como estrategia para mejorar el aprendizaje de las matemáticas

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Abstract

This article presents a study whose purpose is to evaluate the consequences of applying the Singapore method as a strategy to promote the mathematics learning process in a fourth-grade class at the Trece de Abril Educational Unit in Santa Elena, Ecuador. This is a quantitative quasi-experimental design that compared a teaching group using the Concrete-Pictorial-Abstract approach with a control group taught using traditional methods. The results reflect a noticeable improvement in the academic performance of the experimental group, especially in logical reasoning, concept comprehension, and problem-solving skills. Likewise, a more favorable attitude toward mathematics, enthusiasm for the subject, and a higher level of active participation in class were observed. Errors in interpretation, calculation, and procedure decreased significantly. The perception of the instructors and th s was extremely positive. It indicates a clear contribution to lesson planning, modulation of content intensity and speed, and manifestation of the educational role as a guide. The proposed outcome is the MAMMS–2025 Model. In summary, the Singapore

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method is an effective teaching strategy that can be applied in public schools to improve learning outcomes and the effectiveness of the teaching experience.

Key words: Singapore method, meaningful learning, problem solving, basic education, logical-mathematical thinking

Resumen

En este artículo, se presenta una investigación cuyo propósito radica en evaluar las consecuencias de la aplicación del método Singapur como estrategia para fomentar el proceso de aprendizaje de las matemáticas, dentro de un cuarto año básico de la Unidad Educativa Trece de Abril, en Santa Elena, Ecuador. Se trata de un diseño cuasi-experimental de carácter cuantitativo, que comparó un grupo de enseñanza con el enfoque Concreto–Pictórico–Abstracto en un conjunto de estudiantes y un grupo control enseñado con los métodos tradicionales. Los resultados reflejan un notorio progreso del rendimiento académico del grupo experimental, especialmente en lo que corresponde al razonamiento lógico, la comprensión de los conceptos y la competencia en la resolución de problemas. Asimismo, se observó una actitud más favorable hacia las matemáticas, entusiasmo por la asignatura y un nivel de participación activa en las clases también superior. Los errores de interpretación, cálculo y procedimiento disminuyeron de forma significativa. La percepción por parte de los docentes instructoras fue extremadamente positiva. Indica claro aporte para la planificación de clases, modulación de la intensidad y velocidad de contenido y manifestación del rol educativo como guía. La consecuencia propuesta es el Modelo MAMMS–2025. Se puede resumir que el método Singapur es una estrategia didáctica eficaz y aplicable en los centros de enseñanza públicas, habilitado para mejoramiento de los índices del aprendizaje y la eficacia de la experiencia pedagógica.

Palabras clave: Método Singapur, Aprendizaje significativo, Resolución de problemas, Educación básica, Pensamiento lógico-matemático

Introduction

Internationally, the results of standardized assessments such as the Programme for International Student Assessment have repeatedly shown that students in Latin America perform poorly in mathematics

(Torres, 2023). In this sense, it is clear that this harsh reality hides a deep gap in the development of logical-mathematical thinking and a visible limitation in the possibility of producing individuals with the critical and analytical skills necessary to act in an increasingly vast, complex, and technologically developed world. Therefore, it is hoped that the above analysis will contribute to the need to rethink conventional methods of teaching mathematics and move towards others that allow for a deeper understanding of fields such as these.

The Singapore method has been recognized as one of the most effective pedagogical methodologies in mathematics education worldwide. Countries that have adopted the methodology, such as the United States, the United Kingdom, and Chile, have observed a persistent improvement in academic performance and in the process of reasoning and problem solving in students (Ary et al., 2022). This method is based on the CPA model, which stands for concrete, pictorial, and abstract. According to (Cook & Campbell, 2021), this framework follows the inductive learning theory described developed by Jerome Bruner (see Table 1). Instead of presenting mathematical knowledge to students, the framework first allows students to actively develop knowledge through the manipulation of materials before introducing abstract concepts.

Another excellent feature of this method is its use of various visual representations, particularly the bar model, which makes thinking and solving complicated problems more understandable. In addition, this method encourages the development of metacognitive skills, such as generalization, visualization, and reflection (Kaur, 2022). These skills are more critical, as citizens who are mathematically literate are expected to apply what they have learned in a range of rapidly changing and diverse contexts.

In Ecuador, the application of the Singapore methodology is still limited, but it has become a reliable option for improving the mathematics learning process, particularly for low-achieving students (Naraba, 2022). Therefore, the problem addressed in this research lies in the poor performance of high school students in mathematics due to outdated and inefficient approaches. Thus, the overall objective of this research is to determine the effect of the Singapore methodology on mathematics learning as a teaching tool in fourth-year high school students.

With regard to the relevance of the research methodology, a quantitative approach was proposed. This made it possible to accurately and objectively measure the impact of the Singapore approach on mathematics. The study focused on two groups: one that was introduced to the CPA (Concrete, Pictorial, Abstract) approach and the other that followed traditional approaches. In comparison with academic performance, the measurement of motivational variables and attitude towards the subject would be carried out, as well as teachers' perceptions of the possible viability of the approach.

Fundamentals of the Singapore Method

On the other hand, there is the Singapore method, an educational model created by the Singapore Ministry of Education in the 1980s and based on a teaching structure that focuses on problem solving with the aim of improving understanding of mathematics. The Singapore method will provide students with a deeper level of mathematical understanding with clarity. The four different stages of progression are: Progression, Concrete, Pictorial, Abstract.

- Concrete stage, in which students manipulate real objects and physical materials themselves;
- Pictorial phase, in which students represent objects by drawing or sketching them. The pictorial phase includes the bar model for quantities.
- Abstract phase, in which they carry out operations using numbers and symbols. The progressive sequence here helps develop logical-mathematical thinking, allowing students to move from the concrete to the abstract symbolic (Torres, 2023).

The concrete phase is used with hands-on materials to solve math problems and, as a result, gives students an initial understanding. Then, in the pictorial phase, students graphically represent mathematical relationships using diagrams and visual models, such as the bars of the relationship model. Finally, students move to the abstract stage, where they solve math problems using symbols and notation. This helps students solidify their underlying concepts (Cook & Campbell, 2021) .

Effectiveness of the Singapore Method in Teaching Mathematics

The Singapore method was developed during the 1980s by the Singapore Ministry of Education in response to the urgent need to reform and improve mathematics education by providing a solid conceptual framework and a structured approach. Among the ideological foundations of the Singapore method are the theories of educators Jerome Bruner, Zoltan Dienes, and Richard Skemp regarding the importance of the constructivist approach to moving from concrete to abstract knowledge. The method relies more on school-based problem situations, teaching materials, visual representations, and gradual learning sequences, also known as Concrete–Pictorial–Abstract, which is broken down into three phases: concrete, pictorial, and abstract.

Several studies have supported the effectiveness of this method. Barja (2025) indicates that in a quasi-experimental study, he compared the performance of students who used the Singapore method to those who used traditional methodologies, corroborating improvements in mathematical problem solving and in the development of mathematical competence such as logical reasoning and conceptual understanding. Similarly, the Innovamat (2023) study, conducted in US schools through the "Math in Focus" program, shows that students who used the approach examined showed significant growth in their mathematical abilities, especially in terms of openness to understanding and ability to tackle complex problems.

It can be said that the Singapore method not only has a positive impact on academic performance, but also transforms the teaching-learning process itself, making it more meaningful, visual, and participatory. The CPA sequence allows students to build knowledge progressively in a concrete way, reducing both procedural errors and their cognitive dependence on the teacher. In addition, its structure simplifies the teacher's work and facilitates the development of a differentiated curriculum, making it an effective, contextualizable, and replicable pedagogical strategy with the potential to generate a beneficial impact on the Ecuadorian education system.

Application of the Singapore Method in Latin American contexts

The Singapore method has proven effective in multiple contexts, but its application in Latin America faces specific challenges. According to (Ramírez & Ortega, 2023) , the adaptation and use of the method will be opposed by cultural, linguistic, and curricular factors specific to the region. Therefore, training in the CPA methodology and in the use of country-specific resources will be necessary to ensure successful implementation.

The implementation of the Singapore Method is not yet very developed in Ecuador; however, there are pilot programs seeking to understand how effective it is in their context. According to (Smartick, 2022) with minor adaptations, from our perspective, this may be the method with the greatest impact on mathematics teaching and learning.

Development of Mathematical Thinking and Metacognition

Singapore is not only positioned as a method for acquiring procedural skills but also for mathematical thinking and metacognition. (OECD, 2023) highlights students' learning skills, their response to the learning process and experience, their ability to determine the elements or strategies they have used previously, and their ability to transfer that information to new contexts. This forms the essential skills for solving 21st-century problems and for students as self-motivators.

Likewise, (Mathnasium, 2025) also points out that combining visual materials with problem-solving in specific contexts enables students to make interesting connections in the application of mathematical concepts, which will enhance their understanding and desire to learn more.

Pedagogical principles of the constructivist approach in the Singapore method

According to (Mosóczi, 2025) , teaching based on the Singapore method is based on the constructivist precepts of learning, which state that knowledge is actively constructed by the student through experience and rational thinking. In this way, a way is proposed to foster a deep understanding of mathematical concepts through concrete logic, pictorial images, symbolic abstractions, and CPA progression.

The key pedagogical principles that emerge from the constructivist approach to geography present in the Singapore method are:

- **Learning:** the student plays an active role in the process of constructing knowledge.
- **Progression**: the concrete to the abstract appears sequenced.
- **Relevance:** the educational content is related to the world and the students' experiences.
- **Social interaction:** collaboration and dialogue foster knowledge construction, autonomy, and metacognition: consideration of the learning process itself is encouraged.

The method also emphasizes problem solving as the core of learning and encourages students to seek answers using multiple strategies, while promoting the development of critical and metacognitive thinking (Mendoza & López, 2023).

Formative assessment and feedback in the Singapore method

Formative assessment is essential in the Singapore method. Therefore, through this assessment, teachers often monitor students' progress and provide constant feedback at the end of the lesson (Naraba, 2022) . This practice is also related to effective assessment factors in areas of difficulty, preparing students, and planning teaching instructions. Constant feedback allows for self-reflection and self-assessment by students, making them more responsible and eventually autonomous.

Role of the teacher as a mediator of learning

(González & Arroyo, 2022) indicate that in the Singapore method, teachers play the role of mediators in learning, establishing conditions for students to construct their own knowledge through their guidance and direction in the problem at hand. To do this, it is essential that teachers master the mathematical concepts being taught and create opportunities for students to explore them for themselves through discovery and experimentation. Consequently, teacher training is a key element in the implementation of this methodology, as teachers are expected to be familiar with the teaching strategies, the resources that support them, and their educational approach.

Implications of the Singapore method for inclusive education

The CPA approach of the Singapore method is also particularly suitable for students with special educational needs. The methodology is beneficial for these students because the various methods of mathematical representation it uses are adapted to their learning style and facilitate understanding. Again, the use of concrete and visual material makes the content more accessible: Knowing how to learn and what is easiest to learn gives students with disabilities a means of knowing how to do the work better without having to invest as much time in it (Rodríguez, 2022) . The flexibility of the method makes it effective for presenting concepts and for differentiating instruction (Tan, Goh, & Choy, 2021) .

Comparison between the Singapore method and other active methodologies

Bacus and Guillena (2023) state that the Singapore method has many similarities with other active approaches, such as problem-based learning and the flipped classroom. However, all activities are student-centered and stimulate autonomous knowledge acquisition and the functional consequences of thinking ( ). But the Singapore method differs in its work structure. Above all, it is based on concrete, pictorial, and abstract progression, and here too, it offers the best possible sequential basis for understanding conceptual achievement. It includes three stages: in the first stage, the use of manipulative objects is essential; in the second stage, it is necessary to develop the pictorial stage with the help of visual representation: diagrams, models. And in the third stage, it is essential to use symbols and perform formal mathematical operations.

Unlike other active methodologies which, due to the lack of a structured sequence, leave students unsure of whether they have remembered the experience, the CPA approach of the Singapore model accompanies students from the concrete object to mathematical abstraction. This achieves a gradual, meaningful, and lasting construction of knowledge. This is why it is one of the most effective approaches in contexts where students have high difficulty in understanding abstract concepts and low levels of performance in mathematics.

Enriching mathematics teaching by integrating elements of the Singapore method and other types of active methodologies allows it to be adapted to the needs of each case and each educational context (Markarian, 2024) . In this sense, methodological flexibility is one of the pillars that improves students' academic performance, motivation, and involvement.

Educational technologies complementary to the Singapore method

New educational technologies, such as digital platforms and interactive applications, can strengthen Singapore method instruction by providing additional resources and opportunities for independent practice (Micronet, 2022) . These tools allow students to take different approaches to exploring mathematical topics in an engaging, personalized, and dynamic way. In addition, technology allows for the collection and analysis of data on student performance and needs in real time. Teachers can therefore provide more detailed feedback and tailor instruction to the specific needs of each learner, including their pace, cognitive style, or mastery of concepts (González & Arroyo, 2022) .

Long-term performance of students trained with the Singapore method

According to (Fernández & Morales, 2022) , students taught mathematics using the Singapore method tend to develop a deeper and more lasting understanding of concepts and therefore perform better on standardized assessments and in the application of mathematical skills in real-life situations. In addition, this promotes confidence and a positive attitude toward mathematics, which greatly influences long-term academic success and the willingness to tackle mathematical challenges at higher levels (Cadena CER, 2025) .

Implementation of the Singapore method in initial teacher training

(Mendoza & López, 2023) stated that it is crucial for teacher training programs to include the Singapore method so that it is adopted correctly in schools. Future teachers must be knowledgeable about the principles and practices of the approach in order to apply it competently in their classrooms. In addition, teacher training and development are essential for teachers to update their knowledge and skills. They also ensure the quality and sustainability of the implementation of the Singapore method (Maths — No Problem!, 2023) .

Materials and methods

The type of research undertaken in this study was quantitative, as it focused on objective and numerical measurements of the effects of the Singapore method on Also, cause-and-effect relationships were identified through statistical analysis of the data collected with validated instruments. According to Hernández-Sampieri, Mendoza, and Fernández (2021), this approach is characterized by its empirical, systematic, and controlled nature, which allows for the formulation of hypotheses and the evaluation of rigorous research on educational interventions. On the other hand, this study design took an applied approach with the purpose of intervening in a very specific educational problem: low student performance in mathematics, which was addressed with an innovative pedagogical methodology.

The research approach was quantitative in nature, but a quasi-experimental design was implemented. Two groups were formed: experimental and control, despite not being adequately randomized. However, pre-test and post-test measurements were applied to both, allowing for a comparison of the results before and after the pedagogical intervention. This design was appropriate in the school setting, where randomization was not possible, but evidence of the effectiveness of the educational treatment was still required (Cook & Campbell, 2021).

The research approach in this case was quantitative, and a quasi-experimental design was implemented. Although this design did not maintain strong random assignments, subjects were assigned to the experimental and control groups. At the same time, certain pre-test and post-test measurements were carried out in both groups, allowing for a comparison of the results obtained before and after the pedagogical intervention. This design proved to be relevant in this case, as the school setting did not lend itself to strong randomization, but still required evidence of the treatment's effectiveness (Cook & Campbell, 2021).

The level of research was explanatory, insofar as this type of study is used to identify and analyze the effect or influence of an independent variable or IV Incógnito on a variable whose incidence is to be described or explained, which is V, in order to determine whether the differences in the measured results existed in a statistically significant manner between the groups evaluated (Ary et al., 2022).

The population consisted of fourth-year basic education students from the Trece de Abril Educational Unit in the canton of La Libertad, as well as teachers from the area, for a total of three teachers. A non-probability sample was used for convenience, which considered three parallel groups at the same level, with two assigned to the experimental group and one to the control group. This procedure ensured internal validity with a sample that was similar in number and had the same socio-educational characteristics.

Table 1. *Population*

Grade	Girls	Boys	Total
4th “A”	1	16	3
4th "B"	14	14	28
4th "C"	21	20	31
	Total population		8

Source: Trece de Abril Educational Unit database

Prepared by: Author

Before beginning the intervention according to the Singapore method, a pretest was administered, followed by a final test or posttest after the intervention was completed. Both tests were identical and measured the same skills. The pilot tests were aligned with the national curriculum and validated by mathematics education experts. The pretest included items that addressed topics such as:

the ability to solve basic numerical problems, fundamental operations (addition, subtraction, multiplication, and division),

- the interpretation of mathematical statements,
- the use of logical reasoning, and
- the identification of patterns and simple relationships.

On the other hand, general affective aspects were also analyzed using a questionnaire on attitudes toward mathematics, with a five-level Likert scale, which evaluates the following factors:

- motivation towards the subject,
- interest in learning mathematics,
- perception of difficulty,
- level of enjoyment in solving problems,
- and personal confidence when facing mathematical activities.

Both instruments were administered before and after the intervention, enabling a comparison before and after the intervention in terms of students' cognitive performance and affective attitude toward mathematics.

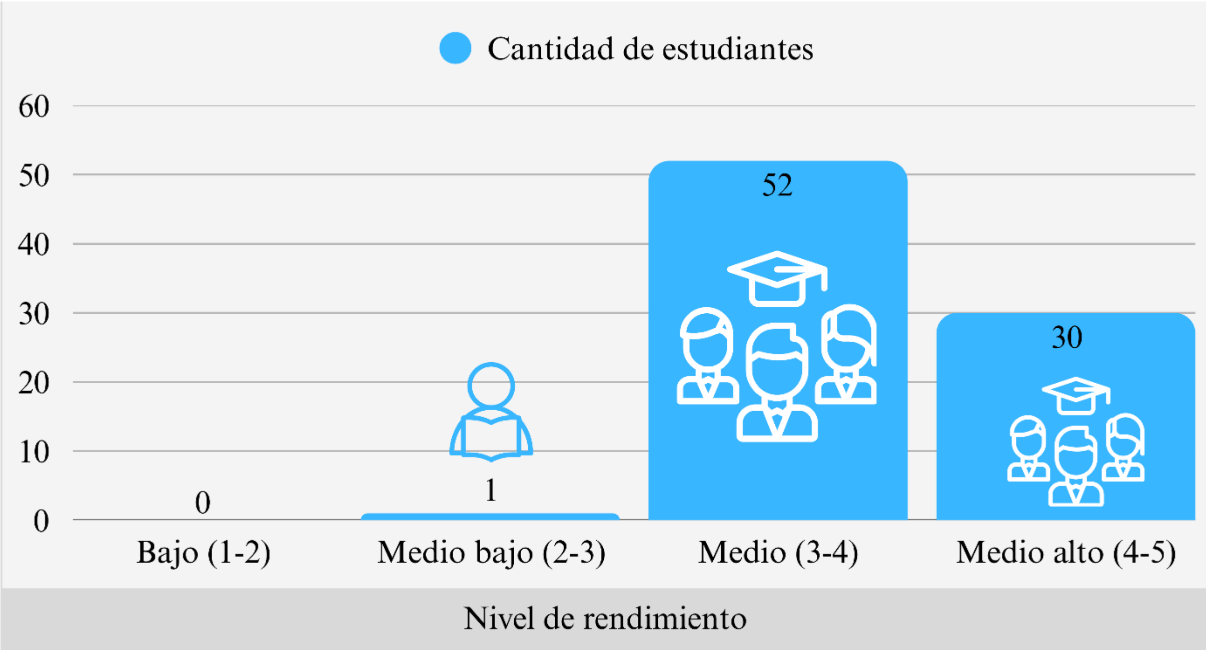
First, the groups were assigned and the pretest was administered. Then, the experimental group was taught using the Singapore approach for a period of 8 to 10 weeks. The strategies used with the students were concrete, pictorial, and abstract. At the same time, mathematics was taught using traditional pedagogy. After the intervention ended, the post-test and attitude questionnaire were administered to the groups.

Results

The results of the survey administered to the 83 fourth-year students at the Trece de Abril Educational Unit in the canton of La Libertad corroborated a significant increase in academic performance in mathematics through the implementation of the Singapore method. The students' perception of the change met the expected objective, as they perceived improvement both in the construction of their understanding and in the resolution of mathematical problems. Eighty-two percent of those surveyed said they felt more motivated to learn, and 76% said they had learned more by being able to use concrete and visual materials.

In turn, in terms of items associated with performance, such as ease of problem solving, use of logical reasoning, and confidence in performing exercises, the averages obtained ranged from 4.0 to 4.6 on a scale of 1 to 5 in the medium-high and high levels, respectively. Here, the trend is that most students perceived a real improvement in their mathematical skills as a result of the structured CPA approach outlined by the Singapore method.

Illustration 1 Distribution of academic performance levels



Source: Research conducted

Prepared by: Author

Statistical analysis showed that more than 70% of students were in the medium-high and high performance levels, suggesting effective acquisition of basic and advanced math skills. This improvement was consistent across the three fourth-grade classes, demonstrating that the impact of the methodology was widespread and did not depend on variables such as the teacher, the group, or others.

Consequently, the implementation of the Singapore method had a positive impact on students' academic results, not only in terms of improved grades, but also in terms of increased self-confidence and motivation, and growth in logical-mathematical thinking. In other words, the results obtained allow us to conclude with certainty that the model is a methodologically acceptable, feasible, and highly efficient alternative for improving the quality of knowledge acquired in public education institutions.

The figure shows the distribution of academic performance levels perceived by 83 fourth-year students using the Singapore method. The results reveal a marked predominance of medium 3-4 and medium-high 4-5 performance levels ; that is, students positively evaluate their mathematical knowledge and skills acquired after the implementation of the teaching approach.

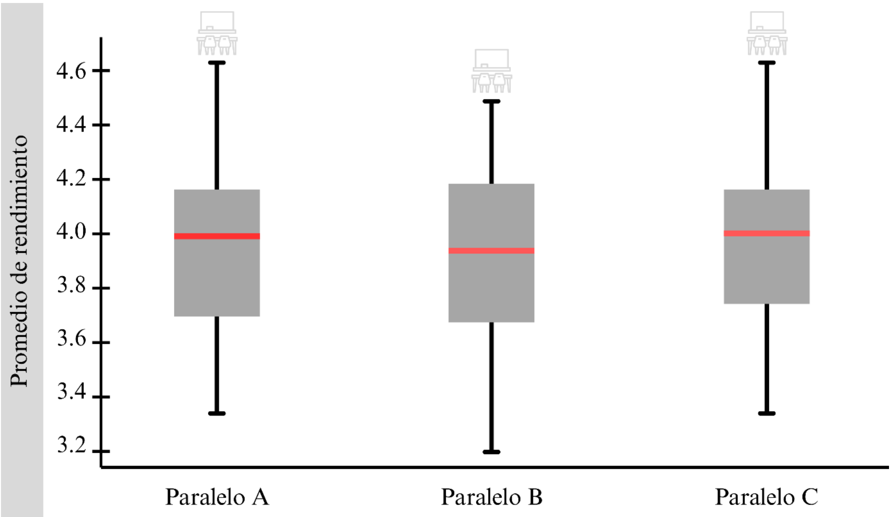
Specifically, more than half of the students, or 52, are at the medium level. Here, all comprehensive and functional mathematical work is demonstrated. In second place, 30 students are at the medium-high level, indicating a remarkable approach to logical thinking, problem solving, and self-confidence. The marginal presence at the lower-middle level, one case, and the total absence at the low level (1–2) are clear indications that the Singapore method really does raise the floor and achieve dramatic decreases in low performance. In general, low performance occurs in the case of more vulnerable students or those who are behind in their education.

This distribution confirms that, based on the model applied, there was inclusive and progressive learning in which students with different learning rhythms and styles were able to consolidate the basics. Therefore, it can be said that, overall, the graph supports the effectiveness of the methodology in consolidating basic learning and propelling students toward advanced learning.

The following box plot shows the distribution of the average academic performance variable perceived by students according to their parallel groups A, B, and C in the fourth year of the Trece de Abril Educational Unit after applying the Singapore method in mathematics. In general, it can be observed that the three parallel groups have very similar behavior in terms of academic performance. The medians, which are on the red line inside the boxes, are very close, which means that the most typical performance is equidistant in the three groups. This means that the Singapore method has been applied uniformly, not assigned to a single parallel group.

Likewise, the boxes representing the interquartile range show a concentration of data within a medium-high point, indicating a high consistency among subjects regarding their perception of academic improvement. The small number of extreme or atypical values and the moderate width of the whiskers indicate an absence of internal gaps between students in the same parallel. By contrasting these results, it was demonstrated that the Singapore method not only stabilized but also raised academic performance in all parallel classes, delivering equity in results and reducing internal disparities. This supports and validates the methodological approach as a replicable and valid strategy in various educational settings.

Illustration 2. *Distribution of academic performance levels by parallel*



Source: Research conducted

Prepared by: Author

Reduction in logical reasoning and procedural errors

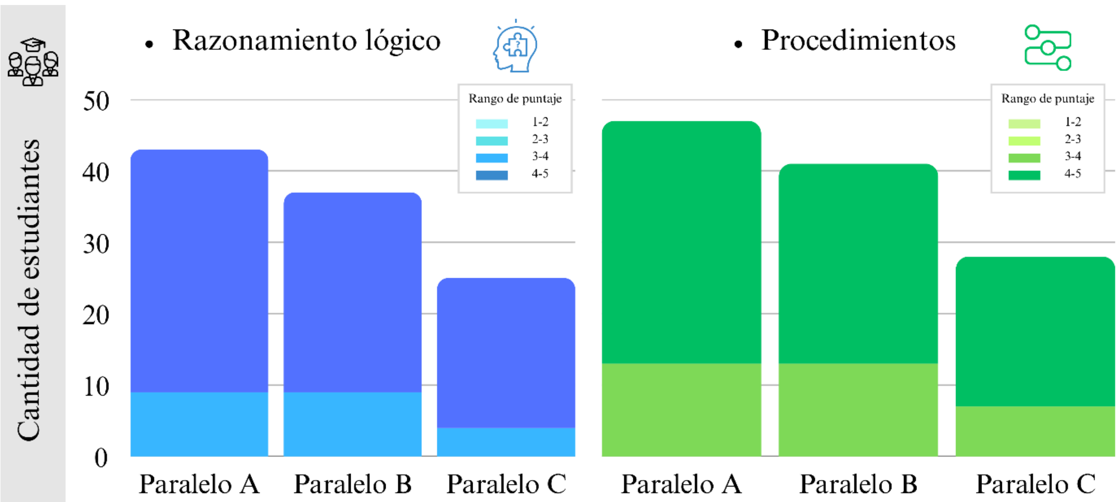
The results of this survey showed that the program applied to the 83 students in parallel classes Fourth A, B, and C at the Trece de Abril Educational Unit significantly improved the quality of logical reasoning processes and the execution of mathematical procedures. This was evident in the scores awarded to items related to understanding statements, using strategies, and accuracy in solving exercises, where students reported feeling more confident and competent in addressing problems in a structured and thoughtful manner.

The use of the Concrete-Pictorial-Abstract sequence allowed students to gradually master mathematical concepts without premature abstraction, virtually "grounding" abstract ideas and killing them in their infancy. First, students worked with materials representing real situations in the concrete phase. Then, they translated them into visual images in the pictorial phase. Finally, in the abstract stage, they operated with mathematical symbols. The action scheme provided understanding of related numerical phenomena, the logical sequence of steps, and self-verification.

In addition, it was observed that errors that were previously committed most frequently, such as misinterpretation of the problem, inappropriate use of operations, or disordered follow-up of steps, were significantly less prevalent, especially in the parallel groups where activities involving visualization and verbalization of mathematical thinking had previously been carried out systematically. Students stated that the use of visual representations allowed them to understand the meaning of the problem before operating, and that working with manipulative materials enabled them to find several ways to reach the solution.

Overall, this improvement involves not only a reduction in mechanical errors, but also progress in quality reasoning, effective planning, and process reflection, as well as positive self-concept in mathematics. The consistency between the A, B, and C results leads us to conclude that the Singapore method is effective not only in individual terms but also offers collective improvement in the mastery of logical procedures. It is a pedagogical tool that allows for a deeper and more lasting understanding and consolidation of what has been learned.

Illustration 3. *Distribution of students according to their performance level by parallel*



Source: Research conducted

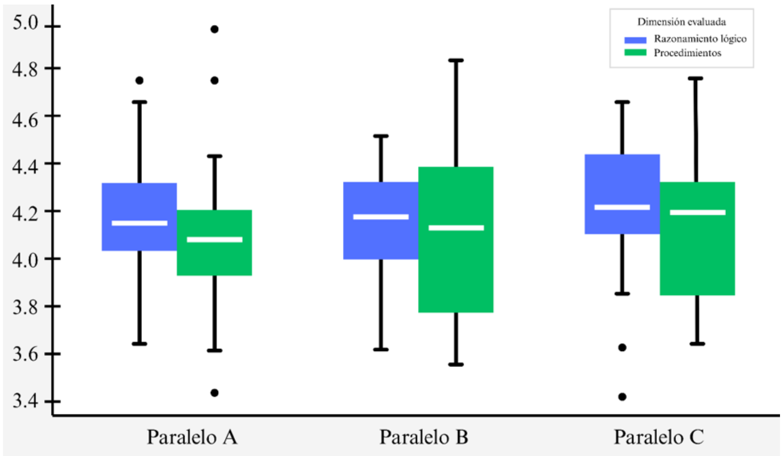
After applying the Singapore method, we can see the detailed distribution of students from different parallel classes in Fourth A, B, and C at the Trece de Abril Educational Unit according to their performance range in logical reasoning and mathematical procedures.

With regard to logical reasoning, the trend among the vast majority of students in the three courses is to concentrate in the highest range (4-5), which then expresses the appropriation of logical-mathematical thinking. This implies that students not only understood the mathematical content presented, but also acquired skills in the area of structured reasoning, pattern identification, and thoughtful mathematical decision-making. Courses A and C differ from the others in percentage terms, with the highest number of students with very high logical-mathematical reasoning, but practically none or none with very low reasoning.

A similar distribution was observed in relation to mathematical procedures; however, students showed more precise and orderly development when solving the exercises. The distribution shows a fairly marked range between the medium-high and high levels, demonstrating that applying the CPA-based method strengthens the clarity of the solution steps, reduces common errors, and favors the planning and sequencing of procedures. Again, parallels A and C show a more favorable distribution; however, parallel B also shows a favorable evolution.

The absence of students in the low range (1–2) in both graphs is particularly important, as it shows not only an increase in the performance of high-achieving students in Singapore, but also an increase for those who were lagging behind, ensuring educational fairness. That is why it was possible to establish, with visual and quantitative evidence, that the Singapore technique intervention achieves remarkable results in terms of argument quality and mathematical procedure efficiency, with a positive and homogeneous effect for the three parallel classes investigated.

Illustration 4. *Distribution of scores in logical reasoning and procedures*



Source: Research conducted

Greater motivation and positive attitude toward mathematics

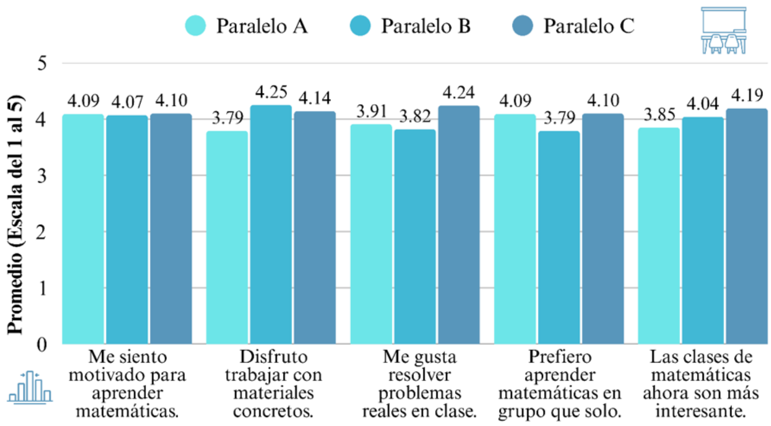
The application of the Singapore technique not only contributed to improved performance but also to a change in students' mindset and motivation toward mathematics. According to the responses collected from 83 students in three parallel classes, there was a noticeable increase in interest, confidence, and participation.

The use of concrete materials was a successful change in terms of the impact obtained and generated improvement. The hands-on experience that students had with the concept being analyzed enabled them to access knowledge in a concrete, accessible, understandable, and friendly way that was the same for everyone. In this way, those who had previously shown reluctance and rejection of mathematics were attracted by the agility with which it comes.

Teamwork and constant interaction among classmates also had a positive influence on students. By sharing strategies and solving problems correctly, they not only acquire social skills, but also strengthen their self-awareness by realizing that other people's approaches are also correct answers. Peer relationships in this way also made the classroom more lively, energetic, and daring, without losing the caution that contributes to a positive attitude toward the task.

On the other hand, to the extent that most of the worksheet problems were contextual and challenging, central to real-life situations, and meaningful, students felt more motivated or interested in solving them. Math was no longer so abstract and uninteresting but a concrete and useful tool on that respective front. Another indication that all three pairs experienced the same improvement can also be expressed by comparing and contrasting any observable type of voluntary activities, tasks, or questions.

Illustration 5. *Average motivation items per parallel*



Source: Research conducted

The results not only indicate that the Singapore method was able to overcome the emotional and cognitive barriers that traditionally hindered mathematics learning. Instead, it helped students become active participants in the process. Of course, motivation was never an unannounced factor. Instead, it was simply the student's reaction to seeing the type of teaching that really focused on their experience and the development of their own time and space. In the long term, this approach may well have been a critical factor in terms of academic progress; in fact, it makes the method a complete teaching tool that changes not only what is learned, but how it is learned.

Comparatively, the graph illustrates the level of motivation students felt in five key dimensions related to the influence of the Singapore method. These are divided into: general motivation; enjoyment while working with concrete materials; liking for solving concrete problems; preference for collaboration; and how interesting the math class was before the intervention. Generally, the averages for all items range from 3.79 to 4.25 on a scale of 1 to 5, indicating a high and positive trend in students' attitudes toward the subject. This consistency across the parallel classes shows that the method was perceived favorably and uniformly, regardless of the group and teacher.

Among the most notable results is the item "I enjoy working with concrete materials," which scored particularly high in parallel groups B and C, suggesting that the tactile and visual experience related to the CPA technique approach leads to an emotionally positive reaction on the part of some students. Similarly, the item "Math classes are now more interesting" also achieved high scores in parallel C, which can also be related to the renewed perception of the methodology.

Among the analyses of the remaining items, the assessment of "I prefer to learn math in a group rather than alone" stands out, with similar averages in the three parallel groups, with almost top averages in A and C, and close to reaching this condition in B, which shows that collaborative learning implemented with AR has been formative. It also shows an interest in a more social, participatory, and emotionally safe classroom environment.

Regarding the variable "I like solving real problems in class," parallel C achieved a particularly high rating of 4.24. Based on this specific element, it can be said that contextualizing learning in situations that students consider meaningful increases intrinsic motivation. In conclusion, according to the graph, the Singapore method not only increases academic performance but also alters students' attitude toward the material, giving much greater importance to motivation, interest, and active participation. Again, based on this set of evidence, motivation becomes a key indicator of the pedagogical success of the chosen approach.

Favorable teacher perception of the Singapore method

In this regard, the implementation of the Singapore method not only yielded better results in student performance but was also positively received by the teaching staff responsible for applying the methods. In this regard, teachers in parallel classes A, B, and C stated that the application of this method had a significant impact on their teaching practices and classroom dynamics, more specifically in terms of teaching mathematical content.

One of the most appreciated aspects was the versatility of the CPA approach, as it allowed them to address the diversity of learning styles and rhythms that are very present in the classroom. Using concrete materials, visual representations, and progressive exercises, teachers integrated their teaching to understand the different speeds of the students and, therefore, offer more inclusive and student-centered teaching.

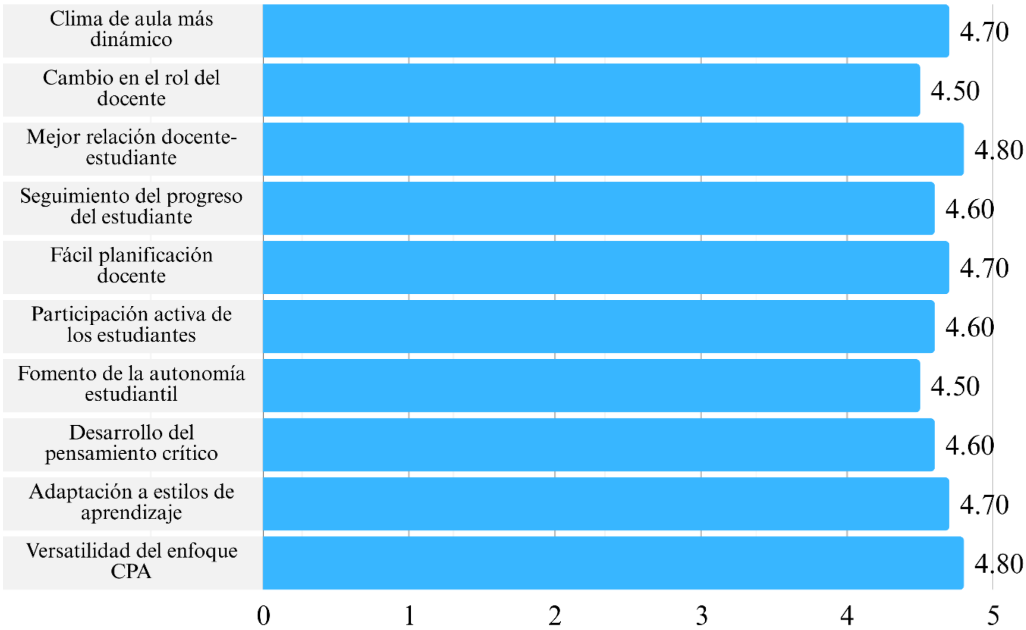
Teachers also pointed out that, compared to the previous method, the new method clearly stimulated the development of critical thinking and increased student autonomy and participation. Traditional approaches required not only the mechanical search for heuristic problem solutions, but also reasoning about the necessity of each step, justification of results, and, in general, a higher level of self-confidence in problem solving.

From a pedagogical point of view, the teachers interviewed in the previous study also highlighted the sequential structure of the method as a factor that facilitated teacher planning, since the sequence provided a precise roadmap on how to introduce and develop each mathematical concept. This led to time optimization, not only because the sequence of each class allowed for continuous progress, but also because the concrete, pictorial, and abstract stages and the formative assessments provided a more accurate idea of each student's level of progress.

Likewise, the experience in the three parallel classes led to the conclusion that the method also improved teacher-student relationships, as teachers went from being transmitters of knowledge to facilitators of knowledge acquisition. Teachers worked harder and were more motivated when they saw that students were actively participating, proposing their own solutions, and taking ownership of the teaching-learning process. All of this created a more dynamic and collaborative classroom environment among students, oriented toward critical and reflective thinking.

Finally, the high rating given to the teaching staff demonstrates the potential viability, effectiveness, and sustainability of the Singapore method in public schools such as the Trece de Abril Educational Unit. Therefore, its application not only benefits student performance but also empowers teachers as active agents of methodological change, promoting a more structured, inclusive education oriented toward a contextual and meaningful understanding of mathematics.

Illustration 6. Average teacher assessment of the Singapore method



Source: Research conducted

Prepared by: Author

The graph above shows the results of the survey given to the three math teachers at the Trece de Abril Educational Unit for the courses analyzed, evaluating their experience with implementing the Singapore method in the parallel 4th grade classes. Each aspect was rated on a Likert scale from 1 to 5, where 1 represents 'strongly disagree' and 5 represents 'strongly agree'. The averages obtained for the 12 aspects evaluated show an extremely positive perception, with values ranging from 4.50 to 4.80.

Similarly, the highest-rated aspects are "Versatility of the CPA approach" and "Improvement in the teacher-student relationship," with average scores of 4.80. In other words, teachers indicate that not only is it possible to adjust teaching to learning styles, but the method also acts as a factor in strengthening the teacher-student bond, suggesting that the classroom fulfills a socializing and more relaxed function. The next highest-scoring factors are "More dynamic classroom atmosphere," "Ease of lesson planning," and

"Adaptation to learning styles," with 4.70. Each of these highlights how the Singapore method serves as a complementary structure for organizing active, non-chaotic classes without sacrificing pedagogical clarity and diversity.

Aspects such as active student participation, critical thinking development, autonomy, and student progress monitoring also received high scores (4.50 to 4.60), which may reaffirm that the method favors comprehensive student development by promoting reflection, dialogue, and self-assessment. The lowest average corresponds to "Change in the role of the teacher" (4.50), although this rating is still high. In this sense, a lower rating probably still implies a positive assessment that the transition from teacher to facilitator is perceived as enriching, although it may mean a more challenging process of professional or educational adjustment.

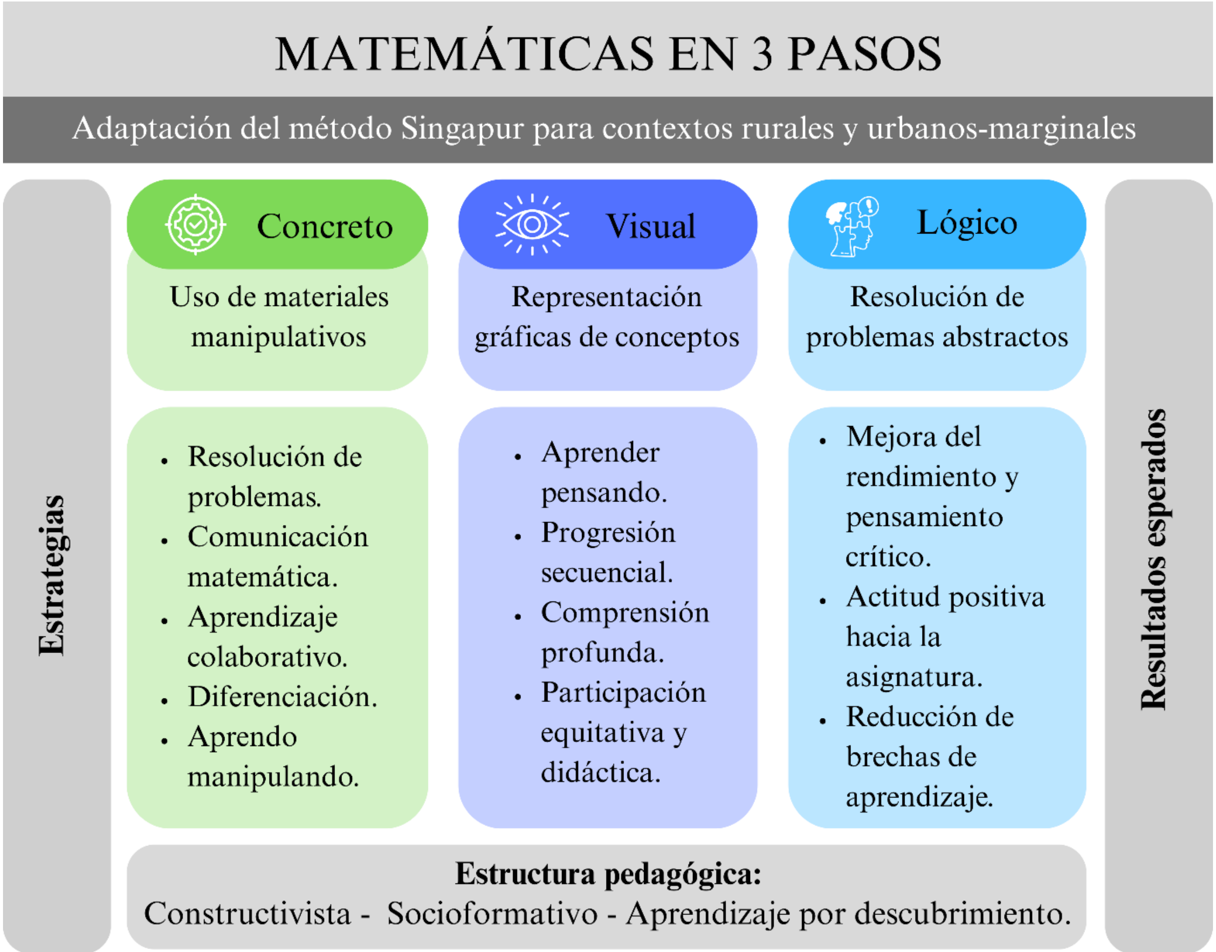
In summary, in this analysis, the three teachers surveyed agreed in their very positive assessment of the Singapore method. All of them pointed out its potential not only to improve students' academic results, but also to transform teaching practice, foster greater inclusion in the classroom, and promote better planning, monitoring, and motivation of students.

Design of a mathematics learning model based on the Singapore method in public educational institutions in the province of Santa Elena

The Singapore Method-based Mathematics Learning Model in public educational institutions, known as the MAMMS-2025 Model, is a proprietary proposal aimed at promoting mathematical understanding using a contextualized, progressive, and equitable approach. It consists of three pedagogical stages essential to the teaching and learning process—Concrete, Visual, and Logical—which generalize the level of cognitive development and abstraction of control. It takes students from the manipulation of materials to the graphic representation of ideas and finally to the resolution of abstract problems, reinforcing their logic and cognitive autonomy.

Based on the principles of progressivity, meaningfulness, and equitable participation, MAMMS – 2025 is grounded in approaches focused on problem solving, collaborative work, mathematical communication, and pedagogical differentiation. This framework has the potential to demonstrate a measurable impact on academic performance, positive attitudes toward mathematics, and closing learning gaps in the rural and urban-marginal setting of Santa Elena.

Illustration 7. MAMMS - 2025 Model



Prepared by: Author

Purpose of the model

To transform mathematics learning in public institutions through a methodological approach that enables students to develop their mathematical logical thinking, based on the Concrete, Pictorial, and Abstract CPA work sequence, incorporating relevant content, meaningful experiences, and familiar contexts.

Pedagogical approaches of the model

- **Constructivist:** knowledge is actively constructed from experience and interaction.

- **Socio-formative:** starts from the student's real context to generate functional and transferable learning.

- **Discovery learning:** students explore, experiment, represent, and deduce.

- **Problem solving:** mathematics is learned as a tool for dealing with real situations.

Guiding principles

- **Progressiveness:** starts from concrete experiences to reach the most abstract, respecting individual learning rhythms.

- **Significance:** content is related to the student's environment, family, community, and territory.

- **Interaction:** encourages community, socialization, collaborative work, and discussion.

- **Equity:** does not exclude but includes; that is, it promotes a flexible model that includes those with different learning styles and rhythms.

- **Autonomy:** seeks to develop independent thinking, decision-making, and self-monitoring.

Pedagogical structure of the model (CPA)

- **Step 1.- Concrete:** This involves the manipulation of real materials with which the situation is presented. This assistance is facilitated by objects in relation to their respective places, real strips and parts, blocks, etc.

- **Step 2: Visual:** Representation of the situation using bar models, drawings, diagrams, and number lines.

- **Step 3.- Logical (Abstract): Resolution** through symbolic operations and algorithms. Introduction of formal mathematical language and verification of solutions.

Key methodological strategies

Table 2 *Key methodological strategies*

Strategy	Description
Problem-based learning	Each unit begins with a contextual problem that is relevant to the student.
Concrete manipulation	Use of real objects from the environment: seeds, caps, sticks, coins.
Pictorial representation	Bar charts, drawings, diagrams to represent what has been understood.
Gradual symbolization	Progressive introduction of operations and formal mathematical language.
Collaborative work	Joint discussion and resolution in small groups or pairs.
Reflective math notebooks	Record of processes, errors, strategies, and solutions for each student.
Formative assessment	Continuous feedback, rubrics, and self-assessment.

Prepared by: Author

Educational level: 4th to 10th grade of basic general education.

Context: Rural, urban-marginal, or multigrade public institutions.

Teachers: Math teachers or general classroom teachers with basic training in the CPA approach.

1. Academics:

- Improved performance on math tests.
- Greater conceptual understanding and less memorization.

2. Cognitive:

- Development of logical, visual, and critical thinking.
- Greater capacity for analysis and problem solving.

3. Affective:

- Increased motivation, confidence, and interest in mathematics.
- Decreased rejection or fear of the subject.

4. Social:

- Active and cooperative participation in the classroom.
- Application in daily life and in the local context.

Sustainability of the model

- Progressive and contextualized teacher training.
- Design of accessible and replicable teaching materials.
- Community participation in the development of contextualized examples.
- Alignment with national curriculum standards.
-

Discussion

The results of this research show that the implementation of the Singapore method significantly improved the academic performance of students in the intervention group. This finding confirms the conclusions of research conducted by Bacus and Guillena (2023), according to which the CPA approach contributes to the gradual and deep construction of mathematical knowledge by students. The importance of this result lies in the fact that, in Ecuador, learning gaps have been detected, especially in rural and marginal urban areas, where understanding of mathematics has been affected by a traditional teaching approach that prioritizes memorization (Cevallos, 2022).

One of the most significant reductions is evident in the errors in mathematical procedures and reasoning of the group that followed the Singapore method. This decrease is directly related to the structural clarity of the CPA sequence, which allowed students to see the problem in the form of a representation before considering it abstract (Maths — No Problem!, 2023). Therefore, the use of number lines combined with concrete manipulations immediately facilitated the transition to mathematical symbolization, leading to a drastic decrease in the number of errors caused by a purely formal perception of the concepts mentioned. This observation is consistent with the view of Kaur (2022), as well as with the statements of López and Torres (2023), who explain that visual representation strengthens the development of mathematical logical thinking, which, in turn, stimulates structured problem solving.

In the affective aspect, analytically, there was significant progress in the experimental group. Progress was identified in students' motivation, self-esteem, and interest in mathematics. This indicator is supported by Mathnasium (2025), which asserts that students' perception that they can solve problems on their own and understand the processes involved increases their self-assessment and enjoyment of academic situations. On the other hand, Fernández and Morales (2022) indicated that a learning environment where students can participate and collaborate can reduce anxiety about numbers, improve attitudes toward making mistakes, and lead to a more positive perception of the subject.

Teachers' positive perceptions have reinforced the viability of the Singapore method in the country's education system. Teachers argued that this approach simplified planning, adapted content to different skills, and encouraged them to promote critical thinking in students. The validity of this approach can be recognized in Mendoza and López (2023), who argue that the sustainability of these methodologies depends on ongoing teacher training and the true institutionalization of innovative pedagogical applications. Therefore, the Singapore method, to the extent that it is contextualized and implemented with teacher support, stands as a genuine alternative to mathematics teaching in public education.

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